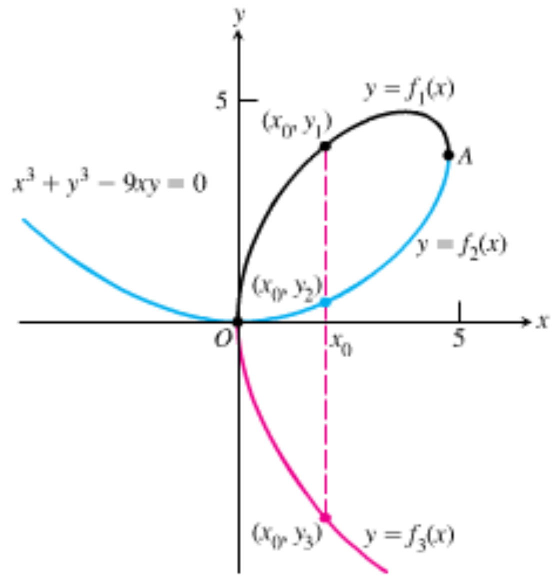
$x^3 + y^3 - 9xy = 0$ (Figure 3.47)




implicit differentiation.

Exploration 2.5:

2. $36 - 24y + 4y^2 + 6 - 12y - 10 = 0$

$4y^2 - 36y + 32 = 0$

$4(y^2 - 9y + 8) = 0$

$4(y-8)(y-1) = 0$

3. $y^2 - 4xy + 4y^2 + x - 12y - 10 = 0$

$2x - (4x\frac{dy}{dx} + y\cdot 4) + 8y\frac{dy}{dx} + 1 - 12\frac{dy}{dx} = 0$

$2x - 4x\frac{dy}{dx} - 4y + 8y\frac{dy}{dx} + 1 - 12\frac{dy}{dx} = 0$

$-4x\frac{dy}{dx} + 8y\frac{dy}{dx} - 12\frac{dy}{dx} = -2x + 4y - 1$

$\frac{dy}{dx}(-4x + 8y - 12) = -2x + 4y - 1$

$\frac{dy}{dx} = \frac{-2x + 4y - 1}{-4x + 8y - 12}$

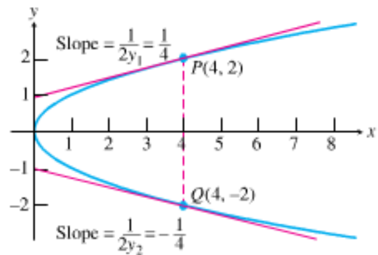
4. $(6,8)$ $(6,1)$

$\frac{-2(6) + 4(8) - 1}{-4(6) + 8(8) - 12}$ $= \frac{19}{28}$

$\frac{-2(6) + 4(1) - 1}{-4(6) + 8(1) - 12}$ $\frac{9}{28}$

**EXAMPLE 1 Differentiating Implicitly**

Find $dy/dx$ if $y^2 = x$.




$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \cdot \frac{dy}{dx}$$

$$2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

**EXAMPLE 2 Finding Slope on a Circle**

Find the slope of the circle $x^2 + y^2 = 25$ at the point $(3, -4)$.

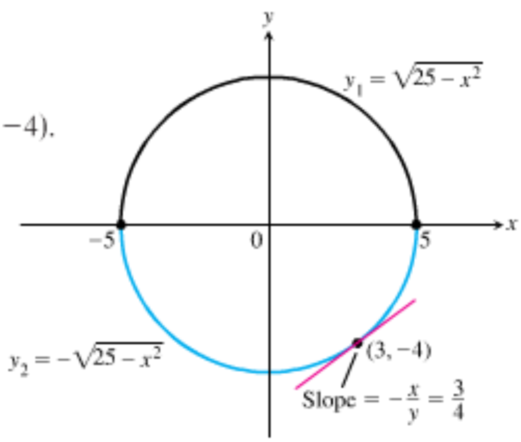



**Figure 3.49** The circle combines the graphs of two functions. The graph of $y_2$ is the lower semicircle and passes through $(3, -4)$. (Example 2)

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

**EXAMPLE 3 Solving for $dy/dx$**

Show that the slope $dy/dx$ is defined at every point on the graph of $2y = x^2 + \sin y$.

$$2\frac{dy}{dx} = 2x + \cos y\frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2 - \cos y) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

$\cos y$ will never $= 2$ $\therefore$ the derivative always exists.

### Implicit Differentiation Process

1. Differentiate both sides of the equation with respect to $x$.
2. Collect the terms with $dy/dx$ on one side of the equation.
3. Factor out $dy/dx$.
4. Solve for $dy/dx$.

**EXAMPLE 4 Tangent and normal to an ellipse**

Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$. (See Figure 3.51.)

$$2x - \left(x\frac{dy}{dx} + y(1)\right) + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

$$= \frac{2-2(-1)}{2(2)-(-1)} = \frac{4}{5}$$ slope of tangent

tangent: $y - 2 = \frac{4}{5}(x+1)$

normal: $y - 2 = -\frac{5}{4}(x+1)$

**EXAMPLE 5** **Finding a Second Derivative Implicitly**

Find $d^2y/dx^2$ if $2x^3 - 3y^2 = 8$.

$$6x^2 - 6y\frac{dy}{dx} = 0$$

$$-6y\frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx} = \frac{-6x^2}{-6y}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2\frac{dy}{dx}}{y^2}$$

$$= \frac{2xy - x^2\left(\frac{x^2}{y}\right)}{y^2}$$

$$= \frac{2xy - \frac{x^4}{y}}{y^2}$$

$$= \frac{2xy^2 - x^4}{y} \cdot \frac{1}{y^2}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

## Rational Powers of Differentiable Functions

**RULE 9 Power Rule for Rational Powers of *x***

If $n$ is any rational number, then

$$\frac{d}{dx}x^n = nx^{n-1}.$$

If $n < 1$, then the derivative does not exist at $x = 0$.

(a) $\frac{d}{dx}(\sqrt{x})$

$= \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

(b) $\frac{d}{dx}(x^{2/3})^{-\frac{3}{3}}$

$= \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

(c) $\frac{d}{dx}(\cos x)^{-1/5^{-\frac{5}{5}}}$

$-\frac{1}{5}\left(\cos x\right)^{-\frac{6}{5}} \cdot -\sin x$

$= \frac{1}{5}\sin x(\cos x)^{-6/5}$